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HEAT TRANSFER IN COMPOSITE MATERIALS DISINTEGRATING UNDER HIGH-RATE ONE-SIDED HEATING

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A mathematical model of heat transfer in heat-protective materials is suggested with the proviso of a square-law temperature dependence of the material density in the zone of thermal destruction of its binder. The influence of certain factors on the experimental temperature field and thermal conductivity of a glass-reinforced epoxy plastic material is shown.

Reliable calculation of temperature fields in disintegrating heat-protective materials (HPM) encounters certain difficulties brought about by various physicochemical processes (e.g., binder destruction) at elevated temperatures and by the dependence of the thermophysical characteristics on both the temperature and the heating rate [1]. Various mathematical models exist to describe heat transfer in such materials [2-4]. In these models thermal destruction of the binder is described by an Arrhenius-type equation (isothermal or nonisothermal). This equation is difficult to use because of the indeterminacy of its parameters (preexponential factor, activation energy, order of the reaction).

In [5] a square-law temperature dependence of the HPM density in the zone of binder destruction is used to describe heat transfer under quasistationary conditions of heating the materials. In the present work this approach is extended to the case of unsteady-state heating and destruction of HPM. This dependence for material density is used in additional terms of the heat conduction equation that take into account heat absorption by the binder during its thermal destruction and by gases produced by the destruction. The influence of the heating rate on binder destruction is accounted for by the temperatures of the initiation and the termination of this process. In [6] it was shown that use of this dependence for material density gives a maximum deviation of +3% in temperature field calculations as compared to the actual dependence $\rho(T)$ in the zone of binder destruction.

When an HPM is subjected to high-rate one-sided heating, three regimes of its heating are observed: I, the heating surface is motionless, the surface temperature is a function of time; II, the heating surface moves according to some law, the surface temperature is a function of time; III, a quasistationary regime (the velocity of the heating surface is constant with a constant temperature of it).

Regime I of HPM heating ($V_w(\tau) = 0$):

$$C_v(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T}{\partial y}\right] \pm q_v; \tag{1}$$

$$\tau = 0,\ T(y,\,0) = T(y);\quad y = 0,\ T(0,\,\tau) = T_w(\tau);\quad y = \infty,\ T(\infty,\,\tau) = T_0; \tag{2}$$

$$q_v = -\Delta H^* \frac{\partial \rho}{\partial T}\frac{\partial T}{\partial \tau} - c_g G_g \frac{\partial T}{\partial y}; \tag{4}$$

$$G_g = \int\limits_{\infty}^{y} \frac{\partial \rho}{\partial \tau} dy = \int\limits_{T_0}^{T} \frac{\partial \rho}{\partial T}\frac{\partial T/\partial \tau}{\partial T/\partial y} dT. \tag{4}$$

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The temperature dependence of the HPM density is as follows:

$$\rho(T) = \begin{cases} \rho_0 , & T_0 \le T \le T_{\text{in}} , \\ \rho_0 \left[1 - \varphi\Gamma \left(\dfrac{T - T_{\text{in}}}{T_{\text{t}} - T_{\text{in}}} \right)^2 \right] & T_{\text{in}} < T < T_{\text{t}} , \\ \rho_0 (1 - \varphi\Gamma) , & T_{\text{t}} \le T \le T_{\text{w}} . \end{cases} \tag{5}$$

Substituting (5) into (3) and (4), we arrive at the following expressions for the power of internal heat sinks:

$$q_{\text{v}} = \begin{cases} 0 , \quad T_0 \le T \le T_{\text{in}} , \\ \dfrac{2\rho_0\varphi\Gamma}{(T_{\text{t}} - T_{\text{in}})^2} \left[\Delta H^* (T - T_{\text{in}}) \dfrac{\partial T}{\partial \tau} + \right. \\ \left. + c_{\text{g}} \dfrac{\partial T}{\partial y} \int\limits_{T_{\text{in}}}^{T} (T - T_{\text{in}}) \dfrac{\partial T / \partial \tau}{\partial T / \partial y} dT \right] , \quad T_{\text{in}} < T < T_{\text{t}} , \\ c_{\text{g}} \dfrac{\partial T}{\partial y} \dfrac{2\rho_0\varphi\Gamma}{(T_{\text{t}} - T_{\text{in}})^2} \int\limits_{T_{\text{in}}}^{T_{\text{t}}} (T - T_{\text{in}}) \dfrac{\partial T / \partial \tau}{\partial T / \partial y} dT , \quad T_{\text{t}} \le T \le T_{\text{w}} . \end{cases} \tag{6}$$

Regime II of HPM heating ($V_{\text{w}}(\tau) = f(\tau)$). Use of the moving coordinate system

$$\begin{cases} x = y - S(\tau) , \\ \tau' = \tau , \end{cases} \quad S(\tau) = \int\limits_{\tau_0}^{\tau} V_{\text{w}}(\tau)\, d\tau ,$$

leads to the following heat conduction equation:

$$C_{\text{v}}(T) \left[\frac{\partial T}{\partial \tau} - V_{\text{w}}(\tau) \frac{\partial T}{\partial x} \right] = \frac{\partial}{\partial x} \left[\lambda(T) \frac{\partial T}{\partial x} \right] \pm q_{\text{v}} ; \tag{7}$$

$$\tau = \tau_0 , \; T(x, \tau_0) = T(y, \tau_0) ; \quad x = 0 , \; T(0, \tau) = T_{\text{w}}(\tau) ; \quad x = \infty , \; T(\infty, \tau) = T_0 . \tag{8}$$

In the general case, the initial distribution for the direct heat conduction problem (DHCP) in this regime of HPM heating is the temperature field obtained from the solution of the DHCP for regime I at the moment τ_0. At $\tau > \tau_0$ $V_{\text{w}} \ne 0$. For regime II of HPM heating the power of internal heat sinks is of the form

$$
q_v = \begin{cases}
0\,, \quad T_0 \le T \le T_{\text{in}}\,, \\[2ex]
\dfrac{2\rho_0\varphi\Gamma}{(T_t - T_{\text{in}})^2}\Bigg[\Delta H^*\,(T - T_{\text{in}})\left(\dfrac{\partial T}{\partial \tau} - V_w(\tau)\dfrac{\partial T}{\partial x}\right) + \\[2ex]
\quad + c_g\dfrac{\partial T}{\partial x}\left(\displaystyle\int_{T_{\text{in}}}^{T}(T - T_{\text{in}})\dfrac{\partial T/\partial\tau}{\partial T/\partial x}\,dT - \dfrac{V_w(\tau)}{2}(T - T_{\text{in}})^2\right)\Bigg]\,, \quad T_{\text{in}} < T < T_t\,, \\[2ex]
c_g\dfrac{2\rho_0\varphi\Gamma}{(T_t - T_{\text{in}})^2}\dfrac{\partial T}{\partial x}\left(\displaystyle\int_{T_{\text{in}}}^{T_t}(T - T_{\text{in}})\dfrac{\partial T/\partial\tau}{\partial T/\partial x}\,dT - \right. \\[2ex]
\quad \left. - \dfrac{V_w(\tau)}{2}(T_t - T_{\text{in}})^2\right)\,, \quad T_t \le T \le T_w\,.
\end{cases} \tag{9}
$$

Regime III of HPM heating ($V_w(\tau) = \text{const}$):

$$
- V_w C_v(T)\frac{\partial T}{dx} = \frac{d}{dx}\left[\lambda(T)\frac{dT}{dx}\right] \pm q_v\,, \tag{10}
$$

$$
x = 0\,,\ T(0\,,\ \tau) = T_w\,; \quad x = \infty\,,\ T(\infty\,,\ \tau) = T_0\,, \tag{11}
$$

$$
q_v = \begin{cases}
0\,, \quad T_0 \le T \le T_{\text{in}}\,, \\[2ex]
-\dfrac{\rho_0\varphi\Gamma V_w}{(T_t - T_{\text{in}})^2}\,[2\Delta H^*\,(T - T_{\text{in}}) + c_g\,(T - T_{\text{in}})^2]\,\dfrac{dT}{dx}\,, \quad T_{\text{in}} < T < T_t\,, \\[2ex]
- c_g\rho_0\varphi\Gamma V_w\dfrac{dT}{dx}\,, \quad T_t \le T \le T_w\,.
\end{cases} \tag{12}
$$

Regimes I and II of HPM heating may also be realized in a bounded space, e.g., on a plate. In this case, the temperature of its "cold" surface will also be a function of time. Regime III may be realized only in a semibounded body. Analogously, the problem on heat transfer in an HPM may also be formulated with other boundary conditions.

For calculating the temperature field in regimes II and III of HPM heating there is no need to pass to a moving coordinate system if we employ the well-known method of fictitious regions. According to this method, we solve the problem in a fixed coordinate system, calculate the position of the moving boundary, and specify an infinitely high thermal conductivity to the left of this boundary and the actual material thermal conductivity of the material to the right of it. In this case the relations of regime I are adopted.

In [5], for the quasistationary regime of HPM heating a solution of Eq. (10) with account for (11) and (12) is obtained in the form

$$
x = \frac{1}{V_w}\int_{T}^{T_w}\frac{\lambda(T)dT}{\displaystyle\int_{T_0}^{T} C_v(T)dT + f_1(T)}\,, \tag{13}
$$

where

$$f_1(T) = -\Delta H^* \int\limits_{T_0}^{T} \frac{\partial\rho}{\partial T} dT - c_g \int\limits_{T_0}^{T}\int\limits_{T_0}^{T} \frac{\partial\rho}{\partial T} dT dT . \tag{14}$$

For the square-law temperature dependence of the material density (5) we may write

$$f_1(T) = \begin{cases} 0, \quad T_0 \le T \le T_{\text{in}}, \\ \rho_0\varphi\Gamma \left(\dfrac{T - T_{\text{in}}}{T_{\text{t}} - T_{\text{in}}}\right)^2 [\Delta H^* + (T - T_{\text{in}})\, c_g/3], \quad T_{\text{in}} < T < T_{\text{t}}, \\ \rho_0\varphi\Gamma\, [\Delta H^* + (3T - 2T_{\text{t}} - T_{\text{in}})\, c_g/3], \quad T_{\text{t}} \le T \le T_{\text{w}} . \end{cases} \tag{15}$$

Relation (13) may be used as a model of a standard unsteady-state temperature field for solving the inverse (coefficient) heat conduction problem (IHCP). For this, we must pass to a fixed coordinate system by using the relation $x = y - V_w\tau$. By specifying various y (a depth of embedding the thermocouples), we obtain the time dependences of the temperature in any number of cross sections. The heating rate is varied by means of the linear rate of removal. The integrals in (13) may be calculated by any approximate method, e.g., the trapezoidal approximation method. In doing so, the error of the temperature field calculation depends on the choice of the temperature step, and for small steps (several tens of degrees) it is the hundredths of a percent.

Based on this solution of DHCP, in [6] a procedure is developed for solving the IHCP that makes it possible to restore the temperature dependence of the thermal conductivity of the material tested. To implement it, it is necessary to record the time variation of the temperature in one cross section of the tested sample under steady-state conditions of material destruction (the quasistationary regime of heating). The entire range of thermocouple readings from T_0 to the maximum temperature is subdivided into N intervals in each of which the thermal conductivity of the material is considered to be constant. The relation for thermal conductivity calculation is as follows:

$$\lambda(\overline{T}_{i+1}) = \frac{V_w^2 (\tau_{i+1} - \tau_i)\, C_v(\overline{T}_{i+1})}{\ln\left(1 + \dfrac{C_v(\overline{T}_{i+1})\,(T_{i+1} - T_i)}{\int\limits_{T_0}^{T_i} C_v(T)\, dT + f_1(T_{i+1})}\right)}, \tag{16}$$

where $i = \overline{1,N}$; $\overline{T}_{i+1} = (T_i + T_{i+1})/2$.

The linear rate of material removal under steady-state conditions is determined by any known method. The integral in (16) is calculated, e.g., by the trapezoidal method, with the temperature range T_0-T_i being subdivided into any number of intervals. This procedure of solving the IHCP is implemented in the KVAZI program.

The relations given above for the power of heat sinks include the temperatures of initiation and termination of thermal destruction of the HPM binder. They are determined by the extrema of the function $\lambda(T)$ [6].

To elucidate the influence of certain factors on the thermal conductivity of HPMs, we tested epoxy binder-based glass-reinforced plastic samples for the cases of convective (products of kerosene combustion in oxygen) and radiative (a solar energy accumulator) heating. A sample of the material tested was a composite cylinder in which grooves were cut parallel to the heating surface by a 10^{-4} m thick diamond disk. The grooves were spaced at a distance of $\sim 10^{-3}$ m. Butt-welded thermocouples VR 5/20 with a diameter of 10^{-4} m, coated with yttrium dioxide as electrical insulation, were set in these grooves. After installation, the grooves were filled with a quartz powder to decrease thermal resistance to a heat flux. The size of the quartz particles was $(0.63\text{-}1.0)\cdot 10^{-4}$ m.

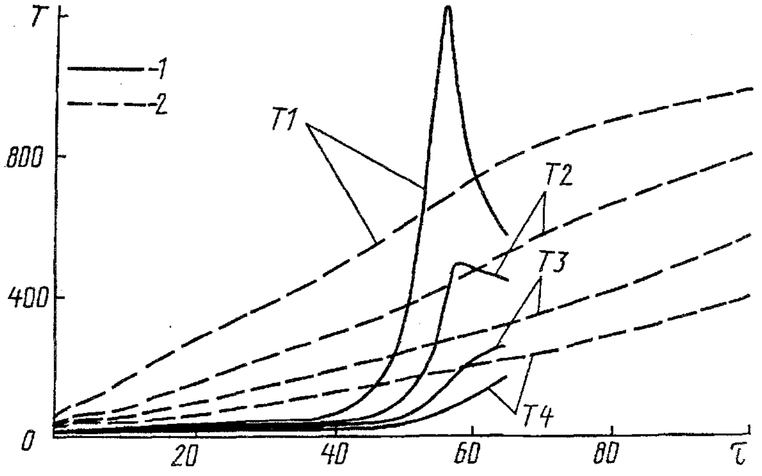

Fig. 1. Temperature fields in samples of glass-reinforced epoxy plastic material: 1, 2, heating by convection and radiation; T1-T4, readings of the I-IVth thermocouples. T, °C; τ, sec.

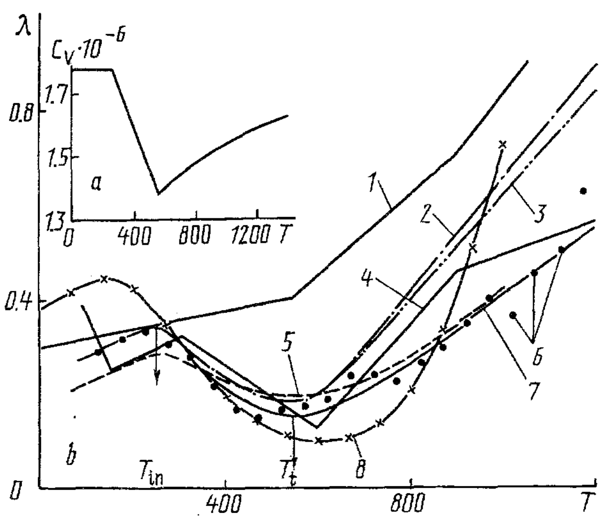

Fig. 2. Thermophysical characteristics of the glass-reinforced epoxy plastic material (a, b, temperature dependences of volumetric specific heat and thermal conductivity, respectively): 1, [12] ($q_V \neq 0$; 2, 3, 6, 7, KVAZI (convective heating: $q_V \neq 0$, $C_V = \text{const}$; $q_V \neq 0$, $C_V = f(T)$; $q_V = 0$, $C_V = \text{const}$; approximation of points 6, correspondingly); 4, 5, PROTON ($q_V = 0$, $C_V = \text{const}$: convective and radiative heating, respectively); 8, [13] ($q_V = 0$, $C_V = \text{const}$). λ, W/(m·K); C_V, J/(m^3·K); T, °C.

The results of determination of temperature fields in the epoxy glass-reinforced plastic samples investigated under convective and radiative heating conditions are shown in Fig. 1. Equal spacing of curves T1 and T2 for the sample subjected to convective heating is indicative of a quasistationary heating regime under these conditions. The maximum heating rate (according to the readings of the first thermocouple from the heating surface) was ~170 deg/sec while the stationary linear rate of removal of the material was $\sim 2 \cdot 10^{-4}$ m/sec. In the case of radiative heating no linear removal of the material was observed and the maximum heating rate recorded by the first thermocouple was ~13 deg/sec. Thus, Fig. 1 shows temperature fields in the epoxy glass-reinforced plastic samples that pertain to regimes I and III of HPM heating.

These temperature fields were processed in the KVAZI and PROTON programs, implementing methods of solution of IHCP. PROTON is a program developed by P. G. Krukovskii (Institute of Theoretical and Technical Physics, Academy of Sciences of the Ukraine) that adopts a procedure based on an interpretation of the method

[7] for solution of IHCP. For its implementation the temperature (as a function of time) must be measured in 3-4 cross sections of the sample of the material tested.

The results of using KVAZI for the case of convective heating of the glass-reinforced plastic material are represented in Fig. 2b by points 6 and approximating curve 7. The values of $\lambda(T)$ obtained using PROTON (linear spline; broken line 4, Fig. 2b) are close to this curve. The difference is within the limits of the error (30-35%) in determining $\lambda(T)$ using these procedures. These data on the thermal conductivity of the glass-reinforced plastic material are obtained for constant volumetric specific heat and zero power of internal heat sinks. In determining $\lambda(T)$ using KVAZI the readings of one thermocouple (T1 and T2) were used but in the case of PROTON four thermocouples were used.

The temperature field in the investigated glass-reinforced plastic under radiative heating was processed in the PROTON program. In doing so, we used both a linear and a cubic spline for representing $\lambda(T)$. The differences between these representations are insignificant in the present case and practically coincide with curve 5 in Fig. 2b.

A comparison of $\lambda(T)$ obtained for the cases of radiative (curve 5, Fig. 2b) and convective (curve 7, Fig. 2b) heating shows that the maximum difference between them is 33% at 550°C, which is within the limits of the error in determining this characteristic. The fact that curve 5 is lower than curve 7 at low temperatures is probably due to semitransparency of the glass-reinforced plastic at low temperatures of the heating surface. This is confirmed by an abrupt increase in the thermocouple readings in the first seconds of radiative heating (see Fig. 1, curves 2). This leads to an error of alternating sign in temperature measurements [8]. An increase in the surface temperature initiates thermal destruction of the glass-reinforced plastic binder, whose products prevent penetration of radiation inside the material, and the thermal conductivity of the glass-reinforced plastic under radiative heating at temperatures above 300°C practically coincides with the $\lambda(T)$ obtained for convective heating. A similar result was obtained in [9] for the mean thermal conductivities of coke and the pyrolysis zone of an epoxy glass-reinforced plastic binder.

In Fig. 2b the temperatures of initiation and termination of thermal destruction of the binder found by the procedure of [6] are marked by arrows. The temperatures determined for convective (curve 7) and radiative (curve 5) heating concide and are 250 and 550°C, respectively.

Knowing the values of T_{in} and T_{t}, we calculated the thermal conductivity of the investigated glass-reinforced plastic using the KVAZI program with account for q_{v}. Account for the power of internal heat sinks in the mathematical heat transfer model leads to an increase in the thermal conductivity of this material (curve 2, Fig. 2b). The maximum increase, compared to curve 7, is 65% at 1200°C. But account for the temperature dependence of C_{v} results in some decrease in $\lambda(T)$ values (curve 3, Fig. 2b), which is caused by a decrease of the integral of $C_{\text{v}}(T)$ in relation (16).

For plotting curve 3 in Fig. 2b we used the temperature dependence of the volumetric specific heat of the epoxy glass-reinforced plastic (Fig. 2a), which is obtained as follows. Up to the temperature T_{in} this characteristic was assumed constant and equal to its value at a room temperature. At the temperatures equal to or higher than T_{t} it was calculated by the relation

$$C_{\text{v c}}(T) = \rho_{\text{c}} c_{\text{c}} = \rho_0 (1 - \varphi \Gamma) [c_{\text{C}}(T) \varphi_{\text{C}} + c_{\text{b}}(T) \varphi_{\text{b}}] =$$

$$= \rho_0 [c_{\text{C}}(T) \varphi (1 - \Gamma) + c_{\text{b}}(T)(1 - \varphi)], \quad T \geq T_{\text{c}},$$

derived under the assumption that the binder of the composite polymer does not undergo any physicochemical changes and does not interact with the coke binder and the coke density does not change. For the epoxy glass-reinforced plastic it has been assumed that its coke consists of carbon and silica. Data on the specific heat of these substances are taken from [10, 11]. In the range T_{in}-T_{t}, $C_{\text{v}}(T)$ is considered to be a linear function.

Thus, $\lambda(T)$ of the epoxy glass-reinforced plastic is strongly affected by account for heat absorption in the mathematical heat transfer model by the binder during its thermal destruction and by gases produced by this destruction.

For comparison, Fig. 2b shows the data of other authors on thermal conductivity of the epoxy binder-based glass-reinforced plastic material (broken line 1 and curve 8) [12, 13].

NOTATION

C_v, volumetric specific heat; T, temperature; τ, time; y, x, coordinates in fixed and moving coordinate systems; λ, thermal conductivity; q_v, power of internal heat sinks (sources); ΔH^*, specific heat of material destruction; ρ_0, ρ_c, density of the initial material and coke at room temperature; φ, content of the binder in the material; Γ, gas amplification factor of the binder; c_g, G_g, specific heat and mass flow rate of the gaseous products of binder destruction; T_0, T_w, initial temperature and heating surface temperature, respectively; V_w, linear velocity of the heating surface; T_{in}, T_t, temperatures of initiation and termination of thermal destruction of the binder; φ_C, φ_b, content of carbon and the binder in the coke of the material; c_c, c_C, c_b, specific heat of coke, carbon, and the binder, respectively.

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